Solution sketch 6 - Computational Models - Spring 2012

Partial solutions. Only ideas are given. Your solutions should include correctness proofs and explanations. For any NPC problem, you should also explain why is it in NP, and show that the reduction is polynomial.

1. (a) NPC. Reduction from VC. Given input $G = (V, E), k$ for VC, we construct a set A_i for every $\{v_1, v_2\} \in E$ and let $A_i = \{v_1, v_2\}$. We keep the same k.
   (b) NPC. Reduction from PARTITION. Given input $x_1, ..., x_n$ for PARTITION we let $a_1 = x_1, ..., a_n = x_n$, $b = 2$ and $c = \llcorner(x_1 + ... + x_n)/2\lrcorner$ (to prove correctness differentiate between 2 cases according to the parity of $x_1 + ... + x_n$).
   (c) P. We check all possible sets of 10 variables.
   (d) NPC. Reduction from 3SAT. Given input ψ we add a clause $(z \vee z \vee \overline{z})$ where z is a new variable.
   (e) NPC. Reduction from UHAMPATH. Given input G, s, t, we construct a graph G' by adding two new vertices u_1, u_2 to G and connecting u_1 to s and u_2 to t.
   (f) NPC. Reduction from previous item. Given input $G = (V, E)$, we construct the input $G, |V| - 1$.
   (g) NPC. Reduction from VC. Given input G, k for VC, we construct input G', k' for our problem. G' is obtained from G by adding it a distinct simple cycle of length $2k$. $k' = 2k$.

2. $A_{TM} \notin R$ and hence it cannot be in NPC. It is NP-hard: Let $L \in NP$. Hence, there exists a TM M which decides L. Given an input x for L we construct an input for A_{TM}: $(\langle M \rangle, x)$.

3. Assume that $P = NP$.

   First: $L \in NP \rightarrow L \in P \rightarrow \overline{L} \in P \rightarrow \overline{L} \in NP \rightarrow L \in coNP$.

   Second: $L \in coNP \rightarrow \overline{L} \in NP \rightarrow \overline{L} \in coNP \rightarrow L \in NP$.

4. Assume $L' \in NPC \cap coNP$.

   First: $L \in NP \rightarrow L \leq_p L' \rightarrow \overline{L} \leq_p \overline{L'} \rightarrow \overline{L} \in NP \rightarrow L \in coNP$.

   Second: $L \in coNP \rightarrow \overline{L} \in NP \rightarrow \overline{L} \leq_p L' \rightarrow L \leq_p \overline{L'} \rightarrow L \in NP$. Therefore $NP = coNP$.