Solution sketch 3 - Computational Models - Spring 2012

As always, these solutions might contain mistakes. Please let us know if you find any! Note: *Only questions 1,3,4,6 will be graded.*

1. A possible implementation: Similar to the TM shown in recitation for PAL. The outer bits are switched while zigzaging toward the center of the input word.

2. Not necesarily. Consider any undecidable L, and $L_1 = \phi$, $L_2 = \Sigma^*$.

3. Let $L \in RE$. Let M_L be a TM that accepts L. That is $L(M_L) = L$. Now, a TM that accepts Prefix(L) will do the following: On input x it will use a monotone enumerator for Σ^* to concurrently simulate M_L on all words of the form xy for $y \in \Sigma^*$ (as was shown in the recitation for the TM that accepts $\overline{E_{TM}}$), and will accept if M_L accepts any of the inputs.

4. Not given.

5. The class of languages is R. One direction is trivial, every TM is equivalent a searching TM. We simply don't use the search option by determining $\delta'((q,a)) = (q', a', L, \text{“}None\text{”})$ whenever $\delta((q,a)) = (q', a', L)$, and $\delta'((q,a)) = (q', a', R, \text{“}None\text{”})$ whenever $\delta((q,a)) = (q', a', R)$. For the other direction, assume that $M' = \langle Q, \Sigma, \Gamma, \delta', q_0, q_a, q_r \rangle$ is a searching TM. We show an equivalent (accepts the same strings, rejects the same strings, and does not halt on the same strings) TM M. The idea is that M has more states (one state for each letter, state and direction) which are used to perform the search, and move to the required state in the end of it. We also use # to mark the head of the tape. Let

$$Q' = Q \uplus \{q_0^a | a \in \Sigma\} \uplus \{p, q_0'\} \uplus \{q_{a,q',i} | a \in \Sigma, q' \in Q\} \uplus \{q_q' | q \in Q, i \in \{L, R\}\}$$

$$M = (Q', \Sigma, \Gamma \uplus \{\#\}, \delta, q_0', q_a, q_r)$$

where:

(a) $\delta((q'_0, a)) = (q^a_0, \#, R)$ for every $a \in \Sigma$.

(b) $\delta((q'_0, \sqcup)) = (q_0, \#, R)$.

(c) $\delta((q^a_0, a')) = (q^{a'}_0, a, R)$ for every $a, a' \in \Sigma$.

(d) $\delta((q^a_0, \sqcup)) = (p, a, L)$ for every $a \in \Sigma$.

(e) $\delta((p, a)) = (p, a, L)$ for every $a \in \Sigma$.

(f) $\delta((p, \#)) = (q_0, \#, R)$.

(g) $\delta((q, a)) = (q', a', i)$ whenever $\delta'((q, a)) = (q', a', i, \text{“}None\text{”})$, $i \in \{L, R\}$.

(h) $\delta((q, a)) = (q_{a'',q',i}, a', i)$ whenever $\delta'((q, a)) = (q', a', i, a'')$, $i \in \{L, R\}$.

(i) $\delta((q_{a',q',i}, a)) = (q_{a',q',i}, a, i)$ for every $a, \neq a'$, $a, a' \in \Sigma$, $q' \in Q$, $i \in \{L, R\}$.

(j) $\delta((q_{a,q'',i}, a)) = (q'_{q''}, a, R)$ for every $a \in \Sigma$, $q'' \in Q$, $i \in \{L, R\}$.

(k) $\delta((q_{a,q',L}, \#)) = (q', \#, R)$ for every $a \in \Sigma$, $q' \in Q$.

(l) $\delta((q'_q, a)) = (q, a, L)$ for every $a \in \Sigma$, $q \in Q$.

(easier to do than to understand).

6. Since a TM that may only move its head to the right can't use the tape as a memory device (whatever the content written, it may not be further accessed), such a TM is equivalent to a finite automaton, hence accepting a regular language.