Solution sketch 2 - Computational Models - Spring 2012

As always, these solutions might contain mistakes. Please let us know if you find any! Note: *Only questions 2,3,5,7 will be graded.*

1. L is finite. Every finite language is regular.
2. (a) L_1 is not regular. Can be proved by Myhill-Nerode Theorem. Note that for every $m_1 \neq m_2$, $b^{m_1} \not\sim_{L_1} b^{m_2}$.
   (b) L_2 is not regular. Can be proved by the pumping lemma. For any $l \leq 0$ choose $s = (^l))(^l$. Let $s = xyz$ s.t. $|y| > 0$ $|xy| \leq l$. Choose $i = 2$. $xy^iz \notin L$ (prove it).
   (c) L_3 is regular. Trivial.
   (d) L_4 is not regular. Use the pumping lemma.
3. (a) $L_1 = \mathcal{L}(\langle\{S\}, \{a, b\}, \{S \to aaSbbb, S \to \epsilon\}, S\rangle)$.
   (b) $L_2 = \mathcal{L}(\langle\{S, A\}, \{a, b, c\}, \{S \to XY, X \to aXb|\epsilon, Y \to bYc|\epsilon\}, S\rangle)$.
   (c) L_3 is not context free. Proof by the pumping lemma. Let $l > 0$ be the critical length. Choose $s = 0^l1^l\#0^l1^l$. $|s| \geq l$ and $s \in L_3$. Assume $s = uvxyz$, $|vy| > 0$, $|vxy| \leq l$:
   - If vy contains # then for every $i \neq 1$, $uv^ixy^iz \notin L_3$.
   - If vy is in the first part (before the #) then for every $i > 1$, $uv^ixy^iz \notin L_3$.
   - If vy is in the second part (after the #) then for $i = 0$, $uv^ixy^iz \notin L_3$.
   - Otherwise, vy contains 1's from the first part and 0's from the second part (why?). Then for $i = 2$, $uv^ixy^iz \notin L_3$ (why?).

   (d) L_4 is not context free. Proof by the pumping lemma.
   (e) L_5 is context free. Proved in recitation.
   (f) $L_6 = \mathcal{L}(\langle\{S\}, \{0, 1\}, \{S \to 0S0|1S1|0X1|1X0, X \to 0X|1X|\epsilon\}, S\rangle)$. Prove that $w \in L_6$ iff is of the form $w = uav\bar{a}u^R$ where X derives $v \in \{0, 1\}$ and S derives $uaX\bar{a}u^R$.

4. (a) The regular languages are closed under this operation. Let $A = \langle Q, \Sigma, \delta, q_0, F\rangle$ be a DFA for L. Assume $Q = \{q_0, ..., q_n\}$. We build an NFA, A' for $Inv(L)$ as follows:

   - A' has 3 parts:
     - Part 1: A copy of A with no accepting states.
     - Part 2: For every $1 \le i, j \le n$ we will have an automata B_{ij} which is a copy of an automata for $Reverse(L)$ (how?).
     - Part 3: A copy of A.
   - For every $1 \le i, j \le n$ we will have an ϵ transition from q_i in part 1 to q_j in B_{ij}, and an ϵ transition from q_i in B_{ij} to q_j in part 3.

   Understand why does this gives an NFA for $Inv(L)$.

   (b) The context free languages are not closed under this operation. Take $L = \{a^n b^n c^m d^m | n, m \in \mathbb{N}\}$. L is context free. Assume by way of contradiction that $Inv(L)$ is also context free. Hence, $L' = Inv(L) \cap L(a^* c^* b^* d^*)$ is also context free (why?). But, $L' = \{a^n c^m b^n d^m | n, m \in \mathbb{N}\}$ (why?), and this language is not context free (why?).

5. (a) $\{0^n 1^m | m \le n\}$

   (b) ϕ

   (c) $L_1 = \{a^i b^j c^{2k} b^j a^i | i, j, k \in \mathbb{N}\}$, $L_2 = \{c^n b^n a^n | n \in \mathbb{N}\}$

6. (a) Pump $w = 0^n 1^n 0^n 1^n 0^n 1^n$

   (b)
$$S \to XX | YY | X | Y$$
$$X \to 0X0 | 0X1 | 1X0 | 1X1 | 1$$
$$Y \to 0Y0 | 0Y1 | 1Y0 | 1Y1 | 0$$

   It implies that the context-free languages are not closed under complementation.

7. (a) **Claim:** Let A be a DFA with n states. $|L(A)|$ is infinite iff $\exists w \in L(A),\ n < |w| \le 2n$.
   Proof: If $\exists w. |w| > n$ then as we learned in the pumping lemma, this word can be pumped infinitely and therefore $L(A)$ is infinite. if $L(A)$ is infinite, then there is a word $w \in L(A)$ such that $|w| > n$. The run of this word in A contains a cycle. We remove all cycles from the run and remember one simple cycle c, $|c| \le n$. The run without the cycles give a word $w' \in L(A), |w'| < n$. We start pumping w' with the cycle c and we will eventually get a word in $L(A)$ in the proper length.

This means that given a DFA A, we can run in A all the words w, such that $n < |w| \leq 2n$. If one of the words is accepted, then $L(A)$ is infinite, otherwise - finite.

(b) First we check if $L(A)$ is infinite. If it is, we return $false$. otherwise, we run in A all words of length at most n and count how many are accepted. we return $true$ iff the count is 9,122,009.