Exercise 6 - Computational Models - Spring 2012

By "graph", we mean undirected graph. By "number", we mean natural number.

1. For the following decision problems, determine whether they are in P or in NPC (assuming $P \neq NP$). Prove your answer.

   (a) Input: sets $A_1, ..., A_n$, and a number k.
   Question: does there exist a set of size k, C, such that for every $1 \leq i \leq n$ $A_i \cap C \neq \emptyset$?

   (b) Input: numbers $a_1, ..., a_n$, b, c.
   Question: can $a_1, ..., a_n$ be divided to b sets that the sum of each one $\leq c$?

   (c) Input: a 3CNF formula ψ.
   Question: does there exist an assignment that satisfies ψ and gives $True$ for exactly 10 variables?

   (d) Input: a 3CNF formula ψ.
   Question: do there exist at least two assignments that satisfies ψ?

   (e) Input: graph G.
   Question: does there exist a Hamiltonian path in G (between any pair of vertices)?

   (f) Input: graph G, a number k.
   Question: does there exist a simple path in G of length $\geq k$?

   (g) Input: graph G, a number k.
   Question: are there a Vertex-Cover S in G of size k and an Independent Set, T, of size $k/2$, such that $T \subseteq S$?

2. Is A_{TM} $NP - complete$? is it $NP - hard$? Prove.

3. Prove: if $NP \neq coNP$ then $P \neq NP$.

4. Prove: if there exists a language $L \in NPC \cap coNP$ then $NP = coNP$.