Exercise 4 - Computational Models - Spring 2012

1. Prove or disprove:
   (a) R is closed under complementation.
   (b) RE is closed under complementation.
   (c) RE is closed under intersection.
   (d) $co-RE$ is closed under intersection.
   (e) RE is closed under Kleene star.
2. A function $f : \Sigma^* \to \Gamma^*$ is computable if there exists a TM that halts with $f(x)$ on its tape, when given x as input. For a function $f : \Sigma^* \to \Gamma^*$ define the language $L_f = \{(x, f(x))|x \in \Sigma^*\}$.
   Show that $L_f \in RE \iff f$ is computable
3. Are the following languages decidable? Prove your answers. Don't use Rice Thm.
   (a) $\{\langle M\rangle|L(M) = \phi\}$
   (b) $\{\langle M_1, M_2\rangle||L(M_1)| \leq |L(M_2)|\}$
   (c) $\{\langle M\rangle|$ *there exists an input that* M *accepts in less then 100 steps*$\}$
4. Let $L = \{\langle M\rangle|L(M)$ is Context Free$\}$. Show that $L \notin RE \cup CoRE$
5. Prove or disprove:
   (a) $EMPTY_{DFA} \leq_m ALL_{DFA}$.
   (b) $L(0^*1^*) \leq_m A_{TM}$.
   (c) if $L \in RE$, then $L \leq_m A_{TM}$.
   (d) $\leq_m$ is a transitive relation.
6. (a) Let L be any infinite language in RE. Prove that there exists an infinite subset $L' \subseteq L$ such that $L' \in R$.
   (b) (* Bonus!) Show that for an infinite language L in general (not necessarily in RE) the claim in (a) is not true.