Exercise 3 - Computational Models - Spring 2012

1. A TM *computes a function* $f : \Sigma^* \rightarrow \Sigma^*$ if it halts with $f(x)$ on its tape, when given x as input. Write a 1-tape TM that computes $f(w) = w^R$ (the reverse of $w \in \{0,1\}^*$). Give a formal description, including the transition function.

2. Let L_1 and L_2 be two decidable languages. Let $L_1 \subseteq L \subseteq L_2$. Is L decidable?

3. Define Prefix(L) $= \{x | \exists y$ such that $xy \in L\}$ Prove that RE is closed under Prefix.

4. Write a 1-tape TM that decides (i.e., halts for any input with the correct answer) $L = \{w \in \{0,1\}^* \mid \#_0(w) \neq 2 \cdot \#_1(w)\}$. Describe the way the TM work. There is no need to specify the transition function.

5. A *searching TM* is a TM that has also a search command. Its transition function δ is from $Q \times \Gamma$ to $Q \times \Gamma \times \{L, R\} \times \{\Gamma \cup \text{“}None\text{”}\}$. $\delta((q,a)) = (q', a', L, a'')$ (or R instead of L) means that if the machine is in state q, and the current cell contains a, then it will write a' instead of it, and the head will move left (or right) till it sees a'', or it reaches the end of the tape. Once it stops moving the state of the TM will be changed to q'. The search is optional: $\delta((q,a)) = (q', a', L, \text{“}None\text{”})$ means that we want usual transition, and no search would be done.

   Determine whether the class of languages decided by searching TMs is R. Prove your claims, and give formal specifications of the TMs in your answer.

6. Suppose that $L(M) = L$ where M is a TM that moves only to the right side. Show that L is regular.