Exercise 1 - Computational Models - Spring 2011/12

Notation: We denote by $\#_\sigma(w)$ the number of times the word $\sigma \in \Sigma^*$ is a substring in the word $w \in \Sigma^*$.

1. Present a DFA that accepts each of the following languages over $\Sigma = \{0, 1\}$:
   (a) Σ^*
   (b) $\{0\}\{1\}^*\{0\}$
   (c) $\{\epsilon, 0101\}$
   (d) $\{w |\ w \text{ does not contain } '1001'\}$

2. Let $n \geq 1$, and let L_n be the language of words over $\Sigma = \{0, 1\}$, such that the nth character from the end is 1.
   (a) Present a DFA that accepts L_n. Give a formal description and not a drawing.
   (b) Present an NFA that accepts L_n.
   (c) Present a regular expression for L_n.

3. Give a DFA for the language over $\Sigma = \{a, b\}$ such that each word starts and ends in the different letters. Give a formal proof of correctness of your construction. (Give an invariant for the language of each state, prove the correctness by induction on the word length, complete the proof based on the invariants.)

4. Present an NFA and convert it to a DFA for the following languages over $\Sigma = \{0, 1\}$:
   (a) $\{w |\ w \text{ contains } '11' \text{ or doesn't contain } '101'\}$
   (b) $\{xy |\ \#_0(x) \bmod 2 = 1 \text{ and } \#_1(y) \bmod 2 = 0\}$

5. Present a regular expression for the following languages over $\Sigma = \{0, 1\}$:

   (a) $\{w | \; |w| \text{ mod } 4 = 2\}$
   (b) $\{w | \; w$ contains exactly three '0's$\}$
   (c) The complement of $\mathcal{L}((0 \cup 10 \cup 110)^*(\epsilon \cup 1 \cup 11))$

6. Given that L is a regular language over some alphabet Σ, prove that the following languages are regular:

   (a) $\{xy | \; (x \in L) \; XOR \; (y \in L)\}$
   (b) $\{x_1x_2 \cdots x_k | \; x_1, \ldots, x_k \in \Sigma$ and $\exists y_1, y_2, \cdots, y_{2k} \in \Sigma, x_1y_1y_2x_2y_3y_4 \cdots x_ky_{2k-1}y_{2k} \in L\}$
   (c) $\{xy | \; yx \in L\}$

7. Is $\{w \in \{0, 1\}^* | \; \#_{01}(w) = \#_{10}(w)\}$ regular? Justify.

8. Let L_1 be a regular language over some alphabet Σ. Let L_2 be a language (not necessarily regular) over the same alphabet.
   Prove that $\{xz | \exists y \in L_2, xyz \in L_1\}$ is regular.